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SAP2D DOCUMENTATION (VAX-11/750 VERSION) - 2D LINEAR ELASTIC FINITE ELEMENT PROGRAM

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1.0 INTRODUCTION

SAP2D is a static, linear elastic finite element program for the analysis of 2-D (2-dimensional) structures (plane or axisymmetric). Most of the program was extracted from Wilson's SAP program [1]* for a CDC 6000 series computer [2]. The program was modified, in 1984, to run on a VAX-11/750 Digital computer, and almost all the SAP2D routines are now ASCII 77 Fortran standard.

Initial stresses, temperature effects, simulation of excavation sequences, and arbitrary distributed loading, gravity loading, as well as concentrated force loading, can be handled by this program.

SAP2D is an efficiently programmed set of routines that can economically handle very large 2-D problems.

The global axis is considered to be a right-handed y-z system in the plane case or a right-handed r-z system in the axisymmetric case.

2.0 TYPE OF ELEMENTS

The basic element used is an isoparametric quadrilateral. A triangle is treated as a degenerate quadrilateral. To increase the accuracy of the element, incompatible displacement modes can be added, *i.e.*, displacements need no longer be continuous across element boundaries, although to guarantee convergence displacement discontinuities over element sides must tend to zero as the element size tends to zero. If not desired, these incompatible modes can be suppressed by correctly changing an input parameter.

If SAP2D is used without boundary elements, it is possible only to specify zero displacements in the y (or r) or z global directions. It is not possible to specify non-zero displacement or even zero displacements in other than the global system.

A boundary element is a point (has a 2×2 stiffness matrix) element with only axial stiffness (*i.e.*, acts like a spring)

To define a boundary element it is necessary to specify two nodal points, one node gives the location of the boundary element and the second is only used to define the axial direction of the element. If no convenient direction node already exists then the user will have to create an artificial node. These artificial nodes should be fully constrained so that no equations are assembled for them.

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* Number(s) refers to cited references.

For each boundary element a displacement and an axial stiffness is specified on the input card. The boundary element routine then applies an axial force equal to the product of the specified displacement and the axial stiffness. Now, if the stiffness specified is large compared to the stiffness of the structure at the point then, the calculated displacement at the point will be approximately equal to the specified displacement.

The effect of a spring support can be simulated by letting the specified displacement equal zero and setting the specified axial stiffness equal to the spring stiffness.

3.0 INITIAL STRESSES

The initial stresses are assumed to vary linearly in the direction of z and they are defined as follows:

$$\begin{aligned}
\sigma_{yy}\ (\text{or } \sigma_{rr}) &= \text{AYY} + \text{BYY} \times z \\
\sigma_{zz} &= \text{AZZ} + \text{BZZ} \times z \\
\sigma_{xx}\ (\text{or } \sigma_{tt}) &= \text{AXX} + \text{BXX} \times z \\
\sigma_{yz}\ (\text{or } \sigma_{rz}) &= \text{AYZ} + \text{BYZ} \times z
\end{aligned}$$

Where σ_{yy} (or σ_{rr}), σ_{zz} and σ_{xx} (or σ_{tt}) are the normal stresses, respectively in y (or r), z and x (or t) directions. σ_{yz} (or σ_{rz}) is the shear stress in the yz (or rz) plane. AYY, BYY, AZZ, BZZ, . . . etc. are the coefficients. The definition of axis is shown in Figure 1.

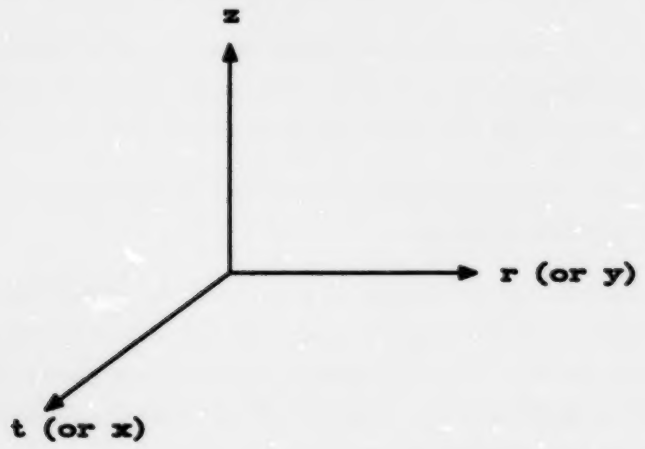


Figure 1. The Definition of Axis.

4.0 TEMPERATURE EFFECTS

Given nodal temperatures and thermal expansion coefficients SAP2D can be used to calculate thermal stresses. Reference temperatures for all elements can be input with the elements. This reference state defines zero thermal stress.

By inputting material properties versus temperature tables the variation of material properties with temperature can be taken into account.

The current version of SAP2D has no facility for inputting nodal temperatures. Normally these would reside on some storage device and would have to be input in the subroutine INPUTJ by suitably modifying the coding from INPUTJ from lines 66 to 68.

5.0 STRUCTURAL LOADING

As well as initial stress and temperature loading as described above it is possible using SAP2D to load the structure by distributed loads (pressures and shears), gravity and concentrated nodal loads.

The major expense in running large SAP2D problems is solving the relevant equations. The major component of this expense is in the decomposition of the stiffness matrix. The additional cost for solving for multiple load vectors is relatively small. To take advantage of this and so reduce the cost SAP2D provides the following procedure for construction of multiple load vectors.

The number of load cases "LL" is input on the control card. For each load case the non-zero concentrated nodal loads are inputed directly. The element forces due to temperature effects, pressure and shear loading, initial stress loads and gravity loads are stored in separate arrays for each element when the element is assembled. For each load case a set of 6 load multipliers is defined. A load vector in then found by adding to the concentrated loads for that load case a linear multiple of the load multipliers and the stored element arrays for each element. Thus, by using various load multiplies, the thermal stress loads, the second the y (or z) component of the distributed loads, the third the z component of the distributed loads, the fourth the initial stress loads, the fifth the y component of the gravity and the sixth the z component of the gravity.

6.0 CUTS

Cuts provide a convenient means for simulating excavation sequences. Suppose an excavation sequence is to be simulated by solving "NPROB" distinct subproblems. By associating with each element a number called a cut number corresponding to the subprob-

lem in which the element is removed (the element remains removed for higher subproblem numbers) the simulation is readily carried out. Note that an element with cut number equal to one would never be used whereas an element with cut number NPROB+1 would never be removed.

7.0 STRESS CALCULATION

By means of a parameter on the element input card the user can control where stresses are calculated for this particular element. Thus, it is possible to calculate no stresses or calculate centroid stresses and/or certain stresses at midpoints of element sides.

The centroid stresses are calculated with respect to the global y-z coordinate system. At a midpoint node the stresses are calculated with respect to a local coordinate system defined by the outward normal of the element side containing the midpoint and the side itself with direction counterclockwise about the element.

As well as calculating the stresses as described above the major and minor principal stresses and their angle are calculated at each stress point. The angle for the centroid stress is with respect to the y-axis and for mid-point stresses is with respect to the outward normal.

8.0 UNITS AND SIGN CONVENTIONS

Any consistent set of units is acceptable, *e.g.*, if co-ordinates are in inches, Young's moduli in psi (pound per square inch), stresses in psi, weight density in pci (pound per cubic inch), then SAP2D will output displacements in inches and stresses in psi. If co-ordinates are in meters, Young's moduli in kPa (kilopascal), stresses in kPa, weight density in kPa/m, then SAP2D will output displacements in meters and stresses in kPa. The continuum mechanics convention, *i.e.*, compressive stresses negative, is adopted. The direction of principal stress is measured from the horizontal axis (y or r-axis), and counter-clockwise positive.

9.0 SAP2D INPUT

9.1 Input Summary

9.1.1 Problem Identification and Control Cards

9.1.2 Nodal Point Input

9.1.3 Plane or Axisymmetric Element Data

(a) Element Parameter Card
(b) Data Defining Material Properties
(c) Element Specification

9.1.4 Boundary Element Data
(a) Element Parameter Card
(b) Boundary Element Specification

9.1.5 Concentrated Nodal Load Input

9.1.6 Load Multipliers

9.2 Detailed Input Description

9.2.1 Problem Identification and Control Cards

(a) Card No. 1 – Title Card
FORMAT (8A10)
READ , HED
HED — An 8 word array where title is stored

(b) Card No. 2 – Control Card
FORMAT (9I5, F10.0)
READ , NUMNP, NELTYP, LL, NPRNT, NPROB, NRES, INDTMP, SCALE

NUMNP — Total number of nodal points.
NELTYP — Number of element types (1 or 2). This is equal to 2 only if boundary elements as well as plane of axisymmetric elements are used. The default value is 1.
LL — Number of load cases, *i.e.*, the number of force vectors that are formed. The default value is 1.
NPRNT — 3 word array print indicator. If non-zero printing is performed. NPRNT(1) controls printing of input data; NPRNT(2) controls printing of displacement output; and NPRNT(3) controls printing of stress output.

NPROB — The number of subproblems. default is 1.

NRES — Initial stress indicator. If non-zero initial stresses are required to input.

INDTMP — Temperature effect indicator. If this parameter is non-zero temperature effects are to be considered by program.

SCALE — Scaling factor for y (or r) and z coordinates. The default value is 1.

Note : Material properties are not scaled by this factor, *i.e.*, material properties must be input using the scaled units.

9.2.2 Nodal Point Input

Each non-generated nodal point has a card associated with it as follows:

FORMAT (3I5, 2E15.7, I5)

READ , N, ID(N,1), ID(N,2), Y(N), Z(N), KN

N — Nodal number.

ID(N,1) — y or r constraint indicator : a zero displacement in y or r direction is indicated if this indicator is non-zero.

ID(N,2) — z constraint indicator : a zero displacement in the z direction is indicated if this indicator is non-zero.

Y(N) — y or r co-ordinate of node N.

Z(N) — z co-ordinate of node N.

KN — Node generate parameter. This is explained more fully in Appendix A.

9.2.3 Plane or Axisymmetric Element Data

(a) Element Parameter Card

FORMAT (6I5)

READ , NPAR(1), NPAR(2), NPAR(3), NPAR(4), NPAR(5), NPAR(6)

NPAR(1) — The number 4.

NPAR(2) — The number of plane or axisymmetric elements.

NPAR(3) — The number of different materials. Default is 1. This is also the maximum material number occurring.

NPAR(4) — Maximum number of temperature cards associated with any material. Default is 1. This is explained more fully below.

NPAR(5) — Structure type indicator. An axisymmetric, plane strain or plane stress structure is indicated by 0, 1 or 2 respectively.

NPAR(6) — Incompatible mode indicator. If non-zero then incompatible modes will be suppressed when element stiffnesses are generated. If the structure is axisymmetic incompatible modes should be suppressed.

(b) Data Defining Material Properties

With each material there is associated $1 + 2 \times$ NTC(MAT) cards.

Card No. 1

FORMAT (2I5, 2F10.0)

READ ,MAT, NTC(MAT), WT(MAT), WANG(MAT)

MAT — Material identification number.

NTC(MAT) — The number of temperature cards defining this material. The default value is 1.

WT(MAT) — The weight density of this material.

WANG(MAT) — The angle of anisotropy with respect to the y or r axis. This is explained more fully on Appendix B.

$2 \times$ NTC(MAT) cards defining the mechanical and thermal properties of this material as a function of temperature.

Each pair of cards is as follows:

FORMAT (8F10.0 / 3F10.0)

READ , TEMP, EN, ES, ET, RNUNS, RNUNT, GNS, HAN, HAS, HAT

TEMP	—	Temperature at which material has the following properties.
EN	—	Young's modulus in N-direction (see Figure 18).
ES	—	Young's modulus in S-direction.
ET	—	Young's modulus in T-direction.
RNUNS	—	Poisson's ratio associated with N-S directions.
RNUNT	—	Poisson's ratio associated with N-T directions.
RNUST	—	Poisson's ratio associated with S-T directions.
GNS	—	Shear modulus in N-S directions.
HAN	—	Coefficient of thermal expansion in N direction.
HAS	—	Coefficient of thermal expansion in S direction.
HAT	—	Coefficient of thermal expansion in T direction.

(c) Data Defining the Initial Stresses

If NRES is non-zero, the initial stresses need to be specified. Otherwise, proceed to (d).

Initial Stress Coefficient Card
FORMAT(8E15.7)
READ ,AYY, BYY, AZZ, BZZ, AXX, BXX, AYZ, BYZ

Where AYY, BYY, AZZ, BZZ, . . . etc. are the coefficients defining the initial stresses as described in Section (3).

(d) Element Specification

Each non-generated element has associated with it 1 + NUMPC input cards (an element card and NUMPC pressure cards associated with the element). Elements must be input in ascending order. Omitted element are generated as described below.

Element Card

FORMAT (10I5, 2F10.0)
READ , M, (IE(I), I=1,4), MAT, NCUT, NUMPC, NS, KG, REFT, THICK

M — The element number.

IE( ) — An array of length 4 which contains the nodal numbers defining the element. These must be in counterclockwise order. If IE(3) = IE(4) or if IE(4) = 0, or blank, then, the element is assumed to be triangular.

MAT — The material number of the element. The default value is 1.

NCUT — The cut number associated with the element. The default value is NPROB + 1 (*i.e.*, the element will never be removed).

NUMPC — The number of element sides that carry distributed pressure and shear loads. Also the number of cards that follow this card and carry distributed load information. The maximum value for NUMPC is 4. (It is assumed that pressure and shears vary linearly over the side of an element. Shear stress is considered positive if it tends to turn an element counter-clockwise.)

NS — Stress point calculation indicator. Stresses can be calculated at the centroid and at mid-points of element sides depending on the value of this parameter. See Figures 2a, 2b and 3 for further information.

KG — Element generation parameter. Default value is 1. This is more fully described in Appendix C.

REFT — The references temperature associated with the element.

THICK — Element thickness. This only has meaning in the plane stress case.

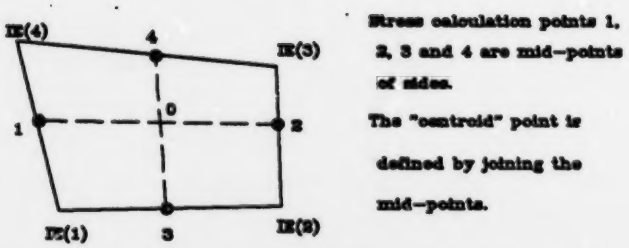


Figure 2a. Stress Calculation Points for a Quadrilateral.

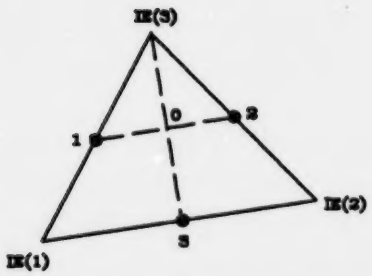


Figure 2b. Stress Calculation Points for a Triangle.

NS	STRESS POINTS
1	none
0	0
8	0, 1
16	0, 1, 2, 3
20	0, 1, 2, 3, 4

Figure 3. Table of NS Value vs Stress Points.

If NUMPC is greater than zero then NUMPC cards follow:

Pressure Cards
FORMAT (I5, 4F10.0)
READ , J, PRES(1,J), PRES(2,J), SHEAR(1,J), SHEAR(2,J)

J	—	The side number of the element. Side IE(1) - IE(2) has side number 1, IE(2) - IE(3) side number 2, IE(3) - IE(4) side number 3 and IE(4) - IE(1) side number 4 assuming element is a quadrilateral.
PRES(1,J)	—	pressure at first node of side J.
PRES(2,J)	—	pressure at second node of side J.
SHEAR(1,J)	—	Shear at first node of side J.
SHEAR(2,J)	—	Shear at second node of side J.

9.2.4 Boundary Element Data

If no boundary elements are specified (*i.e.*, NELTYP on the problem control card equals 1), then, no cards appear hear. Proceed to 9.2.5.

(a) Element Parameter Card
FORMAT (2I5)
READ , NPAR(1), NPAR(2)

NPAR(1)	—	Element type indicator. For a boundary element, NPAR(1) = 7.
NPAR(2)	—	The total number of boundary elements.

(b) Boundary Element Specification

Element cards must be in ascending sequence starting with element 1 and ending with element NPAR(2). Omitted elements are generated as described below:

FORMAT (5I5, 2E15.6)
READ , NEL, NP, NI, KN, NCUT, SD, TRACE

NEL — Element number.
NP — Node number of node where element is located.
NI — Node number of node that with NP defines the axial direction of the boundary element. Direction NP to NI is considered positive. If NI is zero or blank, then, the direction defined by previous input card is used.
KN — Element generation parameter. Default value is 1. This is described more fully in Appendix D.
NCUT — Cut number associated with this element. If zero is taken to be NPROB + 1, *i.e.*, is never removed.
SD — Specified displacement. Note that the direction from location node to direction node is considered positive.
TRACE — Axial stiffness of element. If zero it is taken to be 10^{10}.

9.2.5 Concentrated Nodal Load Input

If no concentrated load data occurs then a blank card must appear here. Only non-zero concentrated nodal load data need be input. Data must appear in non-decreasing nodal number sequence. A node is repeated for each load case with non-zero nodal load at this node. These load cases, for a given node, can be on any order.

FORMAT (2I5, 2F10.4)
READ , N, L, (R(I), I=1,2)

N — The nodal number of node where concentrated force appears.
L — The load case. Possible values of L are from 1 to LL.
R(1) — y or r component of force.
R(2) — z component of force.

9.2.6 Load Multipliers

LL (the number of load cases) cards must appears here. Each card defines one load vector as the sum of the corresponding concentrated load case and as linear combination of the load multipliers and the thermal stress

loads, distributed pressure and the shear loads, initial stress loads and gravity loads respectively.

FORMAT (6F10.0)
READ , ((STR(I,L), I=1,6), L=1,LL)

STR(1,L) — The thermal stress multiplier for load case L.
STR(2,L) — The y or r component multiplier of the distributed load for the load case L. This also multiplies the y or r component of the boundary element loads.
STR(3,L) — The z component multiplier of the distributed load for load case L. This also multiplies the Z component of the boundary forces loads.
STR(4,L) — The initial stress multiplier for the load case L.
STR(5,L) — The y or r component multiplier of the gravity load for the load case L.
STR(6,L) — The z component multiplier of the gravity load for the load case L.

10.0 SAP2D OUTPUT

10.1 Printer Output (Fortran Logical Unit No. 6)

Depending on the values of the printer control parameters, the input data, displacement output and stress output can be printed. Certain additional information *e.g.*, the number of blocks into which the global stiffness matrix is subdivided, the number of equations in a block, the maximum bandwidth, the central processor time spent in various phases of SAP2D, is also printed.

10.2 SAPOUT Output (Fortran Logical Unit No. 10)

In order to facilitate graphical representation of stress and displacement output or carrying out of a stability analysis certain data is written onto an auxilliary storage device (disk or tape). There are $2 + 2 \times$ NPROB (where NPROB is the number of subproblems) files written onto the SAPOUT storage device.

10.2.1 File No. 1 Co-ordinates of Nodal Points

It consists of one record header under FORMAT (1H , 3I10, 8A10). The contents of header are NC, NC1, NUMNP, HED. Here NC = 0, NC1 = 1, NUMNP is the number of nodal points and HED is an eight word vector containing the SAP2D title (the first input card). The rest of the file contains the y and z co-ordinates of all (NUMNP) nodal points.
(Y(I), Z(I), I=1,NUMNP) under FORMAT (1H , 8E15.7).

10.2.2 File No. 2 Element Data

It consists one record header under FORMAT (1H , 3I10, 8A10). The contents of header are NC, NC1, NUME, and HED. Here NC = 0, NC1 = 2, NUME is the number of plane elements (boundary elements are not output) and HED is an eight word title as before. The rest of the file contains NUME records with FORMAT (1H , 7I10). Contents of each record are N, (IX(I), I=1,4), MAT, NCUT, where N is the element number, IX(1), .. , IX(4), the nodal points defining the elements, MAT, the material number of the element and NCUT, the cut number associated with the element.

These 2 files are followed by 2 × NPROB files, *i.e.*, for each subproblem (1 to NPROB) there is associated one file containing the nodal displacements and a second file containing the element stresses.

The Displacement file associated with subproblem NC is as follows:

One record header under FORMAT (1H , 4I10, 8A10). The Contents are NC, NC1, NNP, LL, and HED; where NC is the subproblem number, NC1 = 1, NNP equals the number of non-deleted nodal points, LL is the number of load cases and HED is an 8 word title.
The header is followed by LL × NNP records containing the displacements.
The LL displacement associated with a nodal point are grouped together.
The nodal points with associated displacements are stored in reverse order *i.e.*, the non-deleted nodal point with the largest nodal point number occurs first and non-deleted nodal point with the smallest nodal point number occurs last.

FORMAT (1H , I10, I5, 2E15.7).
Contents of LL records associated with nodes N are:
(N, L, DY(L), DZ(L), L = 1,LL); where N is the nodal point number, L the load number (1 to LL), DY(L) and DZ(L) the y (or r) and z displacements associated

with load case L.

The stress file associated with subproblem NC is as follows:

One record header FORMAT (1H , 4I10, 8A10). The contents are NC, NC1, NUME, LL, HED; where NC is the subproblem number, NC1 = 2, NUME is the number of non-deleted elements, LL is the number of load cases and HED is an 8 word title associated with the problem.

The header is followed by records giving the stresses in each non-deleted element for each load case. With each element it is associated NS × LL records where NS is the number of stress calculation points in the element and LL the number of load cases.

Each record, FORMAT (1H , 2I5, A3, 9E13.7), contents MM, L, STRLAB, (SIG(I), I = 1,7), YC, ZC. STRLAB is a 3 character label defining the stress calculation point (for stress at the centroid STRLAB = 3HCEN). SIG(1), SIG(2), and SIG(3) are the stress components, respectively, in the direction of y (or r), z and x (or t); SIG(4) is the shear stress in the yz (or rz) plane. SIG(5) and SIG(6) are the principal stresses, and, SIG(7) is the angle in degrees that the major principal stress make with the y (or r) axis. The angle is measured, counter-clockwise positive, from the y (or r) axis to the major principal stress. YC and XC are the y (or r) and z co-ordinates of the centroid. For midside locations the stresses come out in local co-ordinates (see stress writeup). No stresses are output for the boundary elements.

11.0 SAP2D FILE USAGE

Three files are used in SAP2D. These are as follows:

Logical File Name	Description
FOR005	The SAP2D input data file;
FOR006	The SAP2D print output file;
SAPOUT	The file containing the co-ordinates of nodal points, element data, and NPROB sets of displacements and stresses data generated by SAP2D.

12.0 SAP2D COMMAND PROCEDURE

The SAP2D command and its sub-command (SAP2) procedure, designed for VAX-11/750 mini-computer operating under VMS 4.1 are given in Appendix E and F. Note that when SAP2D command procedure is used, the two component programs MSHGEN and SAP2DC must be executed prior to SAP2D. Thus, the procedure demands the input filenames for MSHGEN and SAP2DC. The command procedure does the following:

1) The user is prompted for the filename of the MSHGEN input file. The procedure checks that the file exists.

2) The user is prompted for the filename of the SAP2DC input file. The procedure checks that the file exists.

3) The user is prompted for the name of the file to hold the SAP2D output. The user is given the following options:

   (a) A file (*i.e.*, specified by the user either a disk file or a tape file as MSA0: follows by a 6 characters label name) is produced;

   (b) A default file (*i.e.*, SAP2DOUT.DAT) is produced.

4) SAP2D is then submitted to the batch queue and executed.

13.0 SAP2D EXAMPLE

13.1 SAP2D Example with Boundary Elements

An example was devised to illustrate SAP2D input. The structure with its finite element discretization is shown in Figure 4. Note that since roller constraints had to be imposed in directions other than the global directions, it was necessary to make use of boundary elements. The input required for this example is shown in Figure 5.

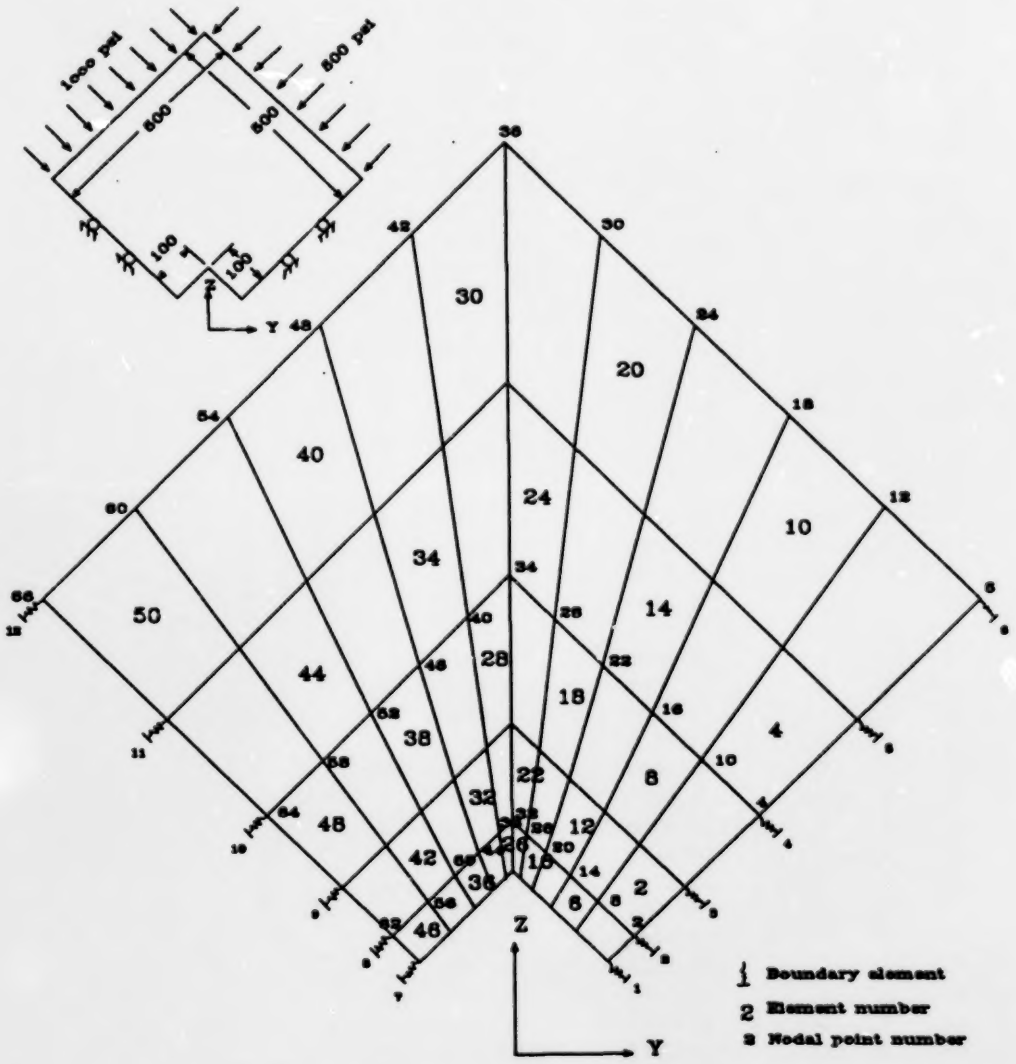


Figure 4. SAP2D Example with Boundary Elements.

Figure 5. Input for SAP2D Example (Figure 4) with Boundary Elements.

```
EXAMPLE - SAP2D WITH BOUNDARY ELEMENTS
   66    2    1    1    1    1    1    0    0      12.0
    1    0    0        70.7110        70.7110    0
    2    0    0        91.0750        91.0750    0
    3    0    0       129.5420       129.5420    0
    4    0    0       186.1100       186.1100    0
    5    0    0       260.7800       260.7800    0
    6    0    0       353.5530       353.5530    0
    7    0    0        47.5180        93.9040    0
    8    0    0        65.6190       116.5300    0
    9    0    0        98.2030       160.8800    0
   10    0    0       145.2670       226.9540    0
   11    0    0       206.8140       314.7530    0
   12    0    0       282.8420       424.2760    0
   13    0    0        28.8500       112.5710    0
   14    0    0        43.7840       138.3640    0
   15    0    0        69.5790       189.5020    0
   16    0    0       106.2350       265.9880    0
   17    0    0       153.7530       367.8200    0
   18    0    0       212.1320       494.9990    0
   19    0    0        14.7080       126.7130    0
   20    0    0        25.5690       156.5770    0
   21    0    0        43.6710       215.4100    0
   22    0    0        69.0130       303.2120    0
   23    0    0       101.5970       419.9820    0
   24    0    0       141.4210       565.7220    0
   25    0    0         5.0910       136.3300    0
   26    0    0        10.9740       171.1700    0
   27    0    0        20.4780       238.6020    0
   28    0    0        33.6020       338.6250    0
   29    0    0        50.3460       471.2390    0
   30    0    0        70.7110       636.4450    0
   31    0    0         0.0000       141.4210    0
   32    0    0         0.0000       182.1430    0
   33    0    0         0.0000       259.0790    0
   34    0    0         0.0000       372.2280    0
   35    0    0         0.0000       521.5910    0
   36    0    0         0.0000       707.1680    0
   37    0    0        -5.0910       136.3300    0
   38    0    0       -10.9740       171.1700    0
   39    0    0       -20.4780       238.6020    0
   40    0    0       -33.6020       338.6250    0
   41    0    0       -50.3460       471.2390    0
   42    0    0       -70.7110       636.4450    0
   43    0    0       -14.7080       126.7130    0
   44    0    0       -25.5690       156.5770    0
   45    0    0       -43.6710       215.4100    0
   46    0    0       -69.0130       303.2120    0
```

- Figure 5 continued -

```
47    0    0       -101.5970        419.9820    0
48    0    0       -141.4210        565.7220    0
49    0    0        -28.8500        112.5710    0
50    0    0        -43.7840        138.3640    0
51    0    0        -69.5790        189.5020    0
52    0    0       -106.2350        265.9880    0
53    0    0       -153.7530        367.8200    0
54    0    0       -212.1320        494.9990    0
55    0    0        -47.5180         93.9040    0
56    0    0        -65.6190        116.5300    0
57    0    0        -98.2030        160.8800    0
58    0    0       -145.2670        226.9540    0
59    0    0       -206.8140        314.7530    0
60    0    0       -282.8420        424.2760    0
61    0    0        -70.7111         70.7110    0
62    0    0        -91.0751         91.0750    0
63    0    0       -129.5421        129.5420    0
64    0    0       -186.1101        186.1100    0
65    0    0       -260.7801        260.7800    0
66    0    0       -353.5531        353.5530    0
 4   50    1    1    1    0
 1    0         0.0          0.0
     0.0 .1000E+07 .1000E+07 .1000E+07      .2000      .2000      .2000 .4176E+06
     0.0         0.0          0.0
 1    1    2    8    7    1    2    0    4    1        0.0        0.0
 2    2    3    9    8    1    2    0    4    1        0.0        0.0
 3    3    4   10    9    1    2    0    4    1        0.0        0.0
 4    4    5   11   10    1    2    0    4    1        0.0        0.0
 5    5    6   12   11    1    2    1    4    1        0.0        0.0
 2  500.0000  500.0000
 6    7    8   14   13    1    2    0    4    1        0.0        0.0
 7    8    9   15   14    1    2    0    4    1        0.0        0.0
 8    9   10   16   15    1    2    0    4    1        0.0        0.0
 9   10   11   17   16    1    2    0    4    1        0.0        0.0
10   11   12   18   17    1    2    1    4    1        0.0        0.0
 2  500.0000  500.0000
11   13   14   20   19    1    2    0    4    1        0.0        0.0
12   14   15   21   20    1    2    0    4    1        0.0        0.0
13   15   16   22   21    1    2    0    4    1        0.0        0.0
14   16   17   23   22    1    2    0    4    1        0.0        0.0
15   17   18   24   23    1    2    1    4    1        0.0        0.0
 2  500.0000  500.0000
16   19   20   26   25    1    2    0    4    1        0.0        0.0
17   20   21   27   26    1    2    0    4    1        0.0        0.0
18   21   22   28   27    1    2    0    4    1        0.0        0.0
19   22   23   29   28    1    2    0    4    1        0.0        0.0
20   23   24   30   29    1    2    1    4    1        0.0        0.0
 2  500.0000  500.0000
```

- Figure 5 continued -

```
21   25   26   32   31    1    2    0    4    1        0.0        0.0
22   26   27   33   32    1    2    0    4    1        0.0        0.0
23   27   28   34   33    1    2    0    4    1        0.0        0.0
24   28   29   35   34    1    2    0    4    1        0.0        0.0
25   29   30   36   35    1    2    1    4    1        0.0        0.0
 2  500.0000  500.0000
26   31   32   38   37    1    2    0    4    1        0.0        0.0
27   32   33   39   38    1    2    0    4    1        0.0        0.0
28   33   34   40   39    1    2    0    4    1        0.0        0.0
29   34   35   41   40    1    2    0    4    1        0.0        0.0
30   35   36   42   41    1    2    1    4    1        0.0        0.0
 2 1000.0000 1000.0000
31   37   38   44   43    1    2    0    4    1        0.0        0.0
32   38   39   45   44    1    2    0    4    1        0.0        0.0
33   39   40   46   45    1    2    0    4    1        0.0        0.0
34   40   41   47   46    1    2    0    4    1        0.0        0.0
35   41   42   48   47    1    2    1    4    1        0.0        0.0
 2 1000.0000 1000.0000
36   43   44   50   49    1    2    0    4    1        0.0        0.0
37   44   45   51   50    1    2    0    4    1        0.0        0.0
38   45   46   52   51    1    2    0    4    1        0.0        0.0
39   46   47   53   52    1    2    0    4    1        0.0        0.0
40   47   48   54   53    1    2    1    4    1        0.0        0.0
 2 1000.0000 1000.0000
41   49   50   56   55    1    2    0    4    1        0.0        0.0
42   50   51   57   56    1    2    0    4    1        0.0        0.0
43   51   52   58   57    1    2    0    4    1        0.0        0.0
44   52   53   59   58    1    2    0    4    1        0.0        0.0
45   53   54   60   59    1    2    1    4    1        0.0        0.0
 2 1000.0000 1000.0000
46   55   56   62   61    1    2    0    4    1        0.0        0.0
47   56   57   63   62    1    2    0    4    1        0.0        0.0
48   57   58   64   63    1    2    0    4    1        0.0        0.0
49   58   59   65   64    1    2    0    4    1        0.0        0.0
50   59   60   66   65    1    2    1    4    1        0.0        0.0
 2 1000.0000 1000.0000
 7   12
 1    1    7
 7   61   55    1
12   66
(A blank card or <CR>)
    0.0        1.0        1.0        1.0        0.0       -1.0
```

13.2 Finite Element Analysis of a 60° Axisymmetric Slope

13.2.1 Some Problems

When carrying out a 2-D finite element analysis the user must choose between approximating the actual geometry either by a plane strain or an axisymmetric model.

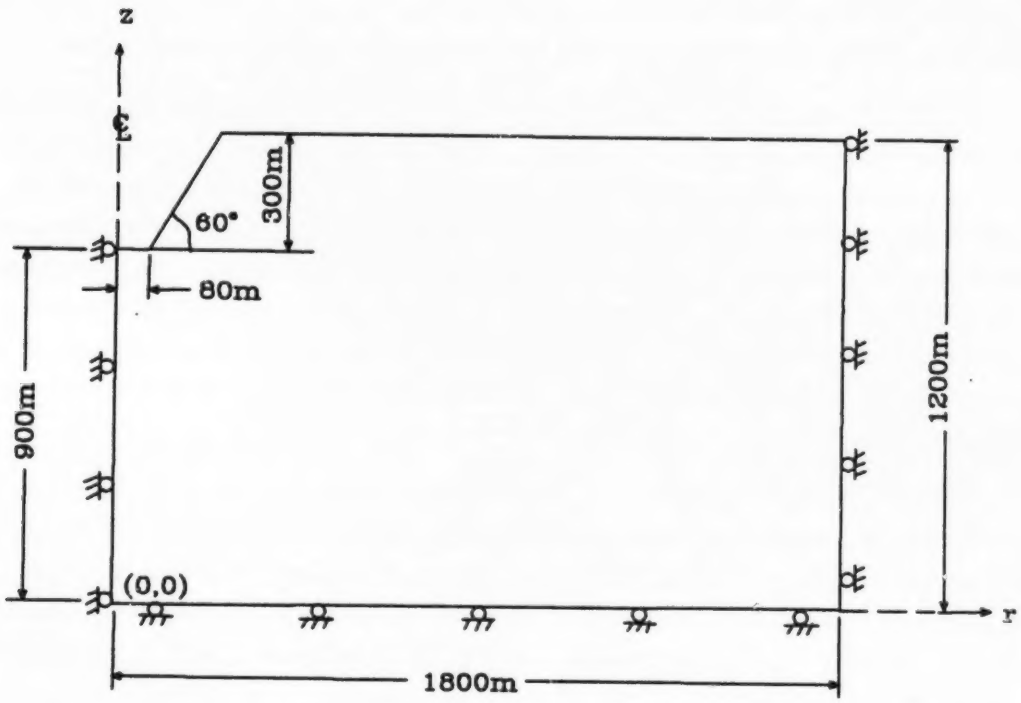
An ever-present problem in modelling mine geometry is the size of the problem. The only reasonable boundary conditions that can be assumed are that the excavation stresses are zero, in the rock mass, at points a large distance from the excavation. However, finite analysis applies only to finite regions and computer costs go up rapidly as we increase the number of nodal points and the number of elements. The user must therefore compromise between finite element error and finite element computer costs. A rule of thumb that has been much used in this regard is that stresses and displacements in the vicinity of the excavation are in error by less than 5% if the fictituous outer boundary is at a distance of 5 times the excavation size.

Another recurrent problem, after assuming that linear elasticity will give at least a first order approximation, is the lack of precise *in situ* values for the elastic constants. This points to one advantage of using stress boundary conditions exclusively. In this case the resulting stress field will be independent of the elastic constants if the rock mass is assumed to be homogeneous. Hence, it is possible to compare a series of designs using arbitrary values for the elastic constants.

The state of initial stress that existed prior to making the excavation is required. This data is particularly difficult to obtain if the rock weight density varies significantly over the structure and a simple analytically defined initial stress field cannot be assumed.

13.2.2 Finite Element Model

The model described here was actually one of a series of models used in a parametric study of the influence of various parameters such as slope angle, slope height and pit width on stresses and displacements in the vicinity of the slope. Figure 6 shows the model chosen for analysis. The extent of the model, the boundary conditions, the material constants and initial stress field are all shown on this figure. Figure 7 gives the zone diagram and Figure 8 the associated key diagram of the model. The definition of zone diagram and that of the key diagram are given in reference [3].



Material constants assumed:

Young's modulus	70.3×10^6 kPa
Poisson's ratio	0.25
Weight density	27 kPa/m (27 kN/m^3)

Initial stress coefficients estimated as following:

AYY = −10800.0 BYY = 9.0;
AZZ = −32400.0 BZZ = 27.0;
AXX = −10800.0 BXX = 9.0;
AYZ = BYZ = 0.0

Figure 6. 60° Axisymmetric Slope with Gravity Loading.

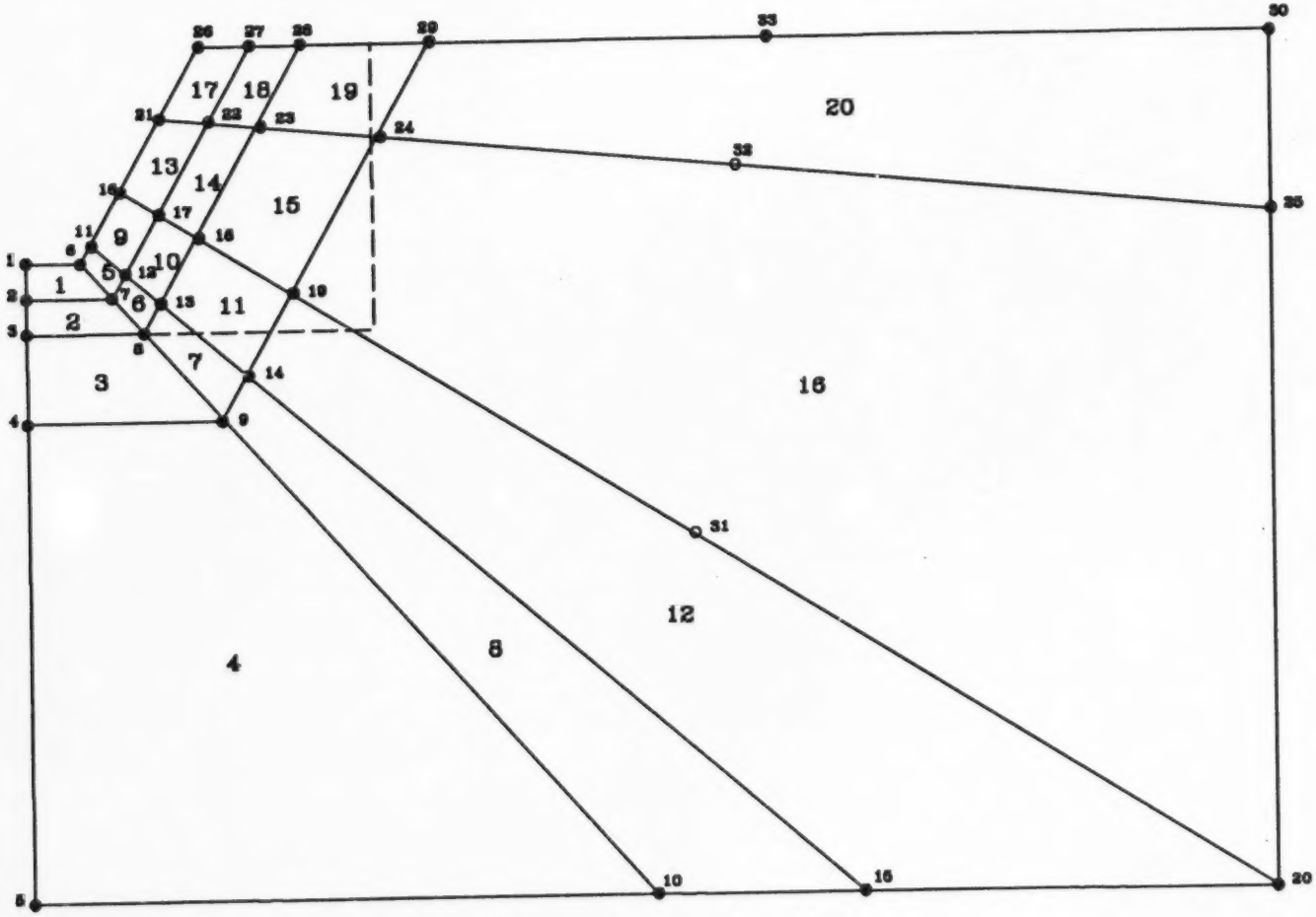


Figure 7. Zone Diagram of 60° Axisymmetric Slope.
(° 's denote specified point, dashed line is boundary of critical region)

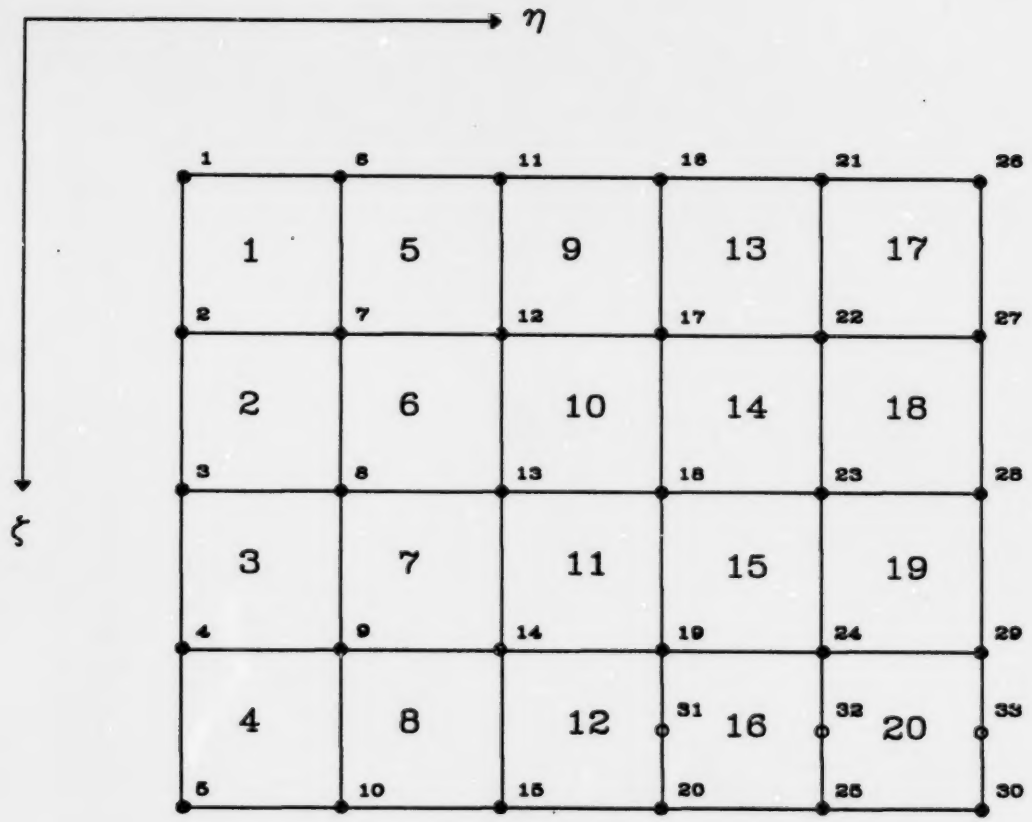


Figure 8. Key Diagram of 60° Axisymmetric Slope.

In Figure 7 the boundary of the region of interest or the critical region (*i.e.*, the region next to the excavation in which we are primarily interested) is denoted by dashed lines. Figure 9 shows the finite element mesh produced by MSHGEN [3] and MSHPLT [4]. There are 1802 nodal points and 1716 elements. The elements in the critical region have been magnified as shown in Figure 10, and the nodal and element numbers are not plotted. This was also produced by MSHPLT [4].

The mesh produced by MSHGEN [3] was merged with additional card input in SAP2DC [5] to produce a SAP2D input file. The resulting stiffness matrix had 3516 equations and a half bandwidth of 71.

13.2.3 Displacements

SAP2D produces tables of nodal point numbers versus r and z-displacements on the printer. Figure 11 shows the first page of printer displacement output for the 60° axisymmtric slope. SAP2D outputs the displacements in reverse nodal point order.

The obvious way of representing displacements in two dimensions is by plotting the displacement as vectors. The displacements as suitable scaled for easy visual inspection. Figure 12 shows an example of such a vector plot of excavation displacement over the critical region.

13.2.4 Stresses

SAP2D prints tables of element numbers versus stress field. The stress field for an axisymmetric structure is made up of σ_{rr}, σ_{zz}, σ_{tt} and σ_{rz}. The principal stresses in the r-z plane are also calculated and printed as well as the angle with respect to the r-axis of the major principal stress (most positive one). Figure 13 shows the first page of printer output for the 60° axisymmetric slope. As well as printing SAP2D stress, the nodal point co-ordinates, element data, the displacements and the stresses are stored on disk or tape for possible future graphical representation.

Figure 14 shows an example of a plot of stress in the critical region of the 60° axisymmetric slope. Vectors are drawn without arrows and compressive stresses are distinguished from tensile stresses by having a bar across the vector. Unfortunately, such a plot can become quite cluttered when the element density is high. An additional drawback is that tensile stresses, which are usually small (although still important) compared to compressive stresses, are difficult to see.

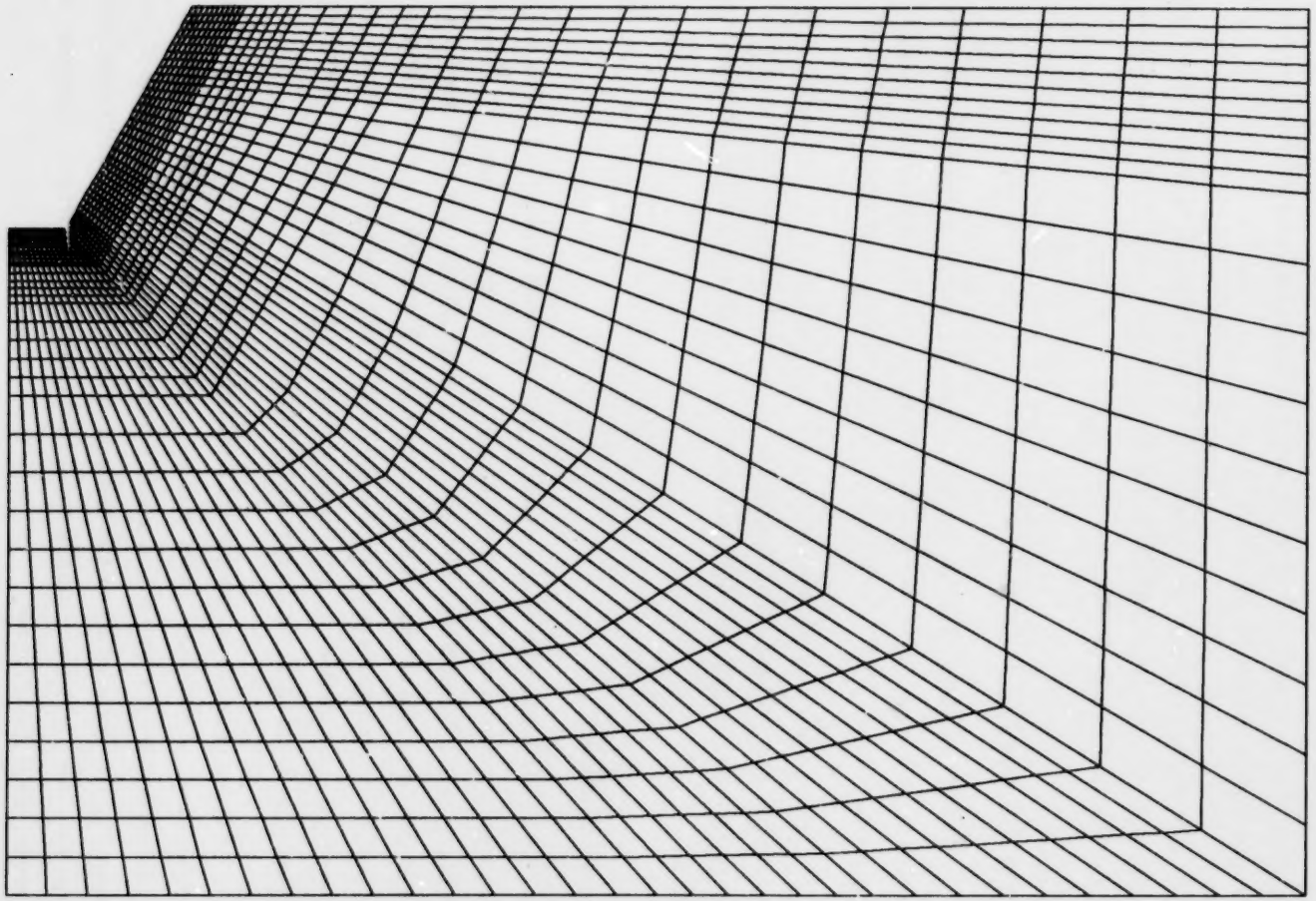


Figure 9. Finite Element Discretization of 60 Degree Axisymmetric Slope. (Model Scale : 1 cm = 92.3 m)

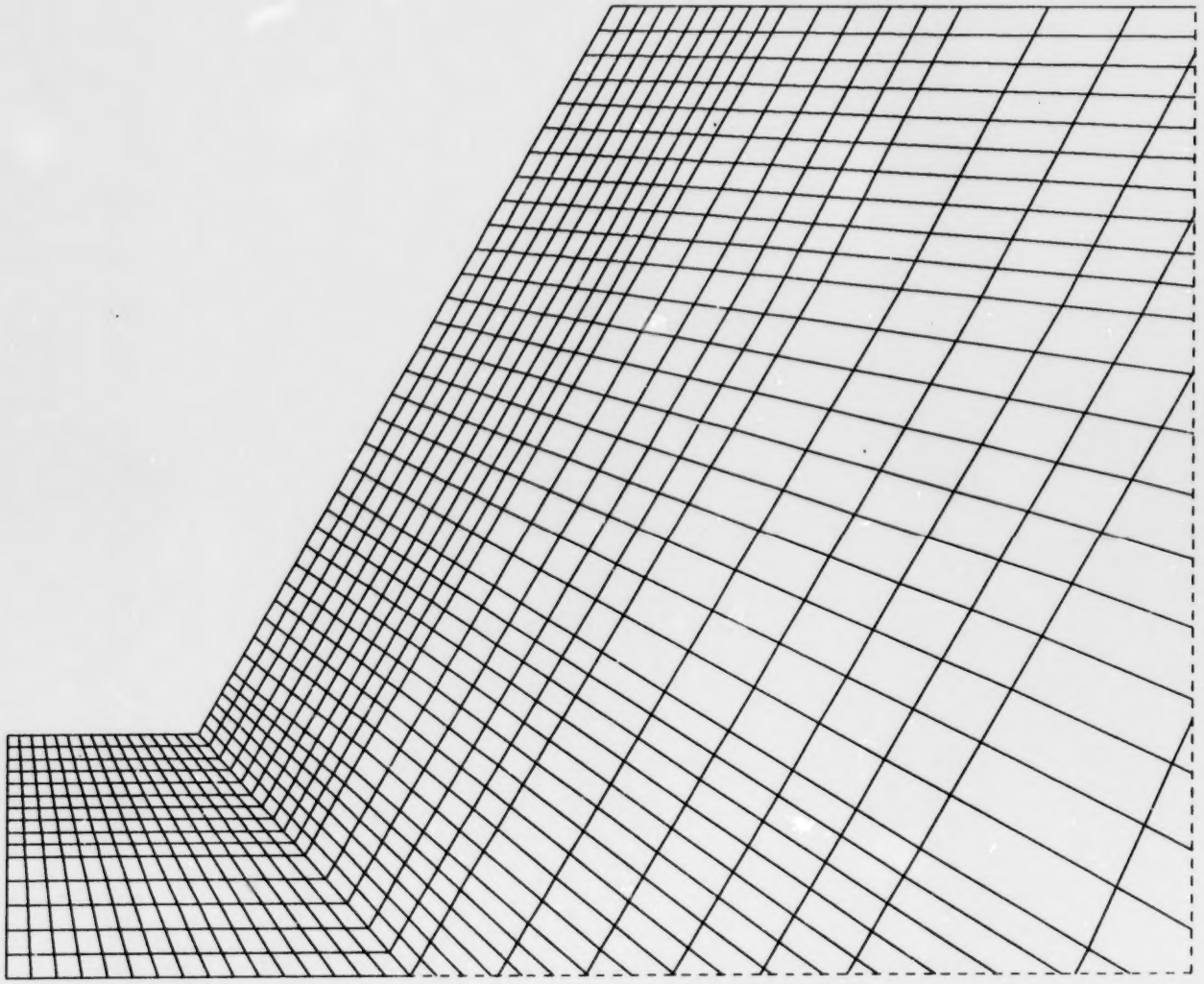


Figure 10. Finite Element Discretization of Critical Region.
(Model Scale : 1 cm = 27.0 m)

NODAL DISPLACEMENTS

NODE	LOAD	Y-DISPLACEMENT	Z-DISPLACEMENT
1802	1	0.0000000E+00	-1.6808076E-02
1801	1	6.1942888E-03	-1.5833728E-02
1800	1	1.2928857E-02	-1.2664695E-02
1799	1	2.0454420E-02	-7.2182785E-03
1798	1	2.9026629E-02	7.2444644E-04
1797	1	3.8866736E-02	1.1478441E-02
1796	1	5.0164398E-02	2.5420953E-02
1795	1	6.3083366E-02	4.2978063E-02
1794	1	7.7762276E-02	6.4606920E-02
1793	1	9.4314136E-02	9.0778768E-02
1792	1	1.1281888E-01	1.2195587E-01
1791	1	1.3330731E-01	1.5856147E-01
1790	1	1.5571943E-01	2.0095006E-01
1789	1	1.7969999E-01	2.4933620E-01
1788	1	1.9692993E-01	2.8439647E-01
1787	1	2.1621613E-01	3.2516831E-01
1786	1	2.3779947E-01	3.7281209E-01
1785	1	2.6193473E-01	4.2896938E-01
1784	1	2.8854248E-01	4.9595538E-01
1783	1	3.0046302E-01	5.2611703E-01
1782	1	3.1289673E-01	5.5885082E-01
1781	1	3.2577464E-01	5.9432107E-01
1780	1	3.3901921E-01	6.3278466E-01
1779	1	3.5250160E-01	6.7458922E-01
1778	1	3.5934752E-01	6.9659507E-01
1777	1	3.6621356E-01	7.1958077E-01
1776	1	3.7304646E-01	7.4356008E-01
1775	1	3.7979257E-01	7.6856130E-01
1774	1	3.8638484E-01	7.9461157E-01
1773	1	3.9273676E-01	8.2172579E-01
1772	1	3.9873224E-01	8.4989387E-01
1771	1	4.0421858E-01	8.7905627E-01
1770	1	4.0899682E-01	9.0904415E-01
1769	1	4.1274324E-01	9.3935567E-01
1768	1	0.0000000E+00	-1.7189221E-02
1767	1	5.8309687E-03	-1.6183754E-02
1766	1	1.2187396E-02	-1.2980425E-02
1765	1	1.9329382E-02	-7.4723451E-03
1764	1	2.7488003E-02	5.6770467E-04
1763	1	3.6871843E-02	1.1471225E-02
1762	1	4.7657985E-02	2.5633169E-02
1761	1	5.9997831E-02	4.3501087E-02
1760	1	7.4016839E-02	6.5557145E-02
1759	1	8.9811794E-02	9.2300996E-02
1758	1	1.0744281E-01	1.2422901E-01
1757	1	1.2691814E-01	1.6180144E-01
1756	1	1.4810909E-01	2.0544936E-01
1755	1	1.7093864E-01	2.5522065E-01
1754	1	1.8692032E-01	2.9109299E-01
1753	1	2.0470168E-01	3.3279833E-01
1752	1	2.2445786E-01	3.8157040E-01
1751	1	2.4628864E-01	4.3917790E-01
1750	1	2.7055663E-01	5.0754178E-01

Figure 11. Sample of SAP2D Printer Displacement (cm) Output.

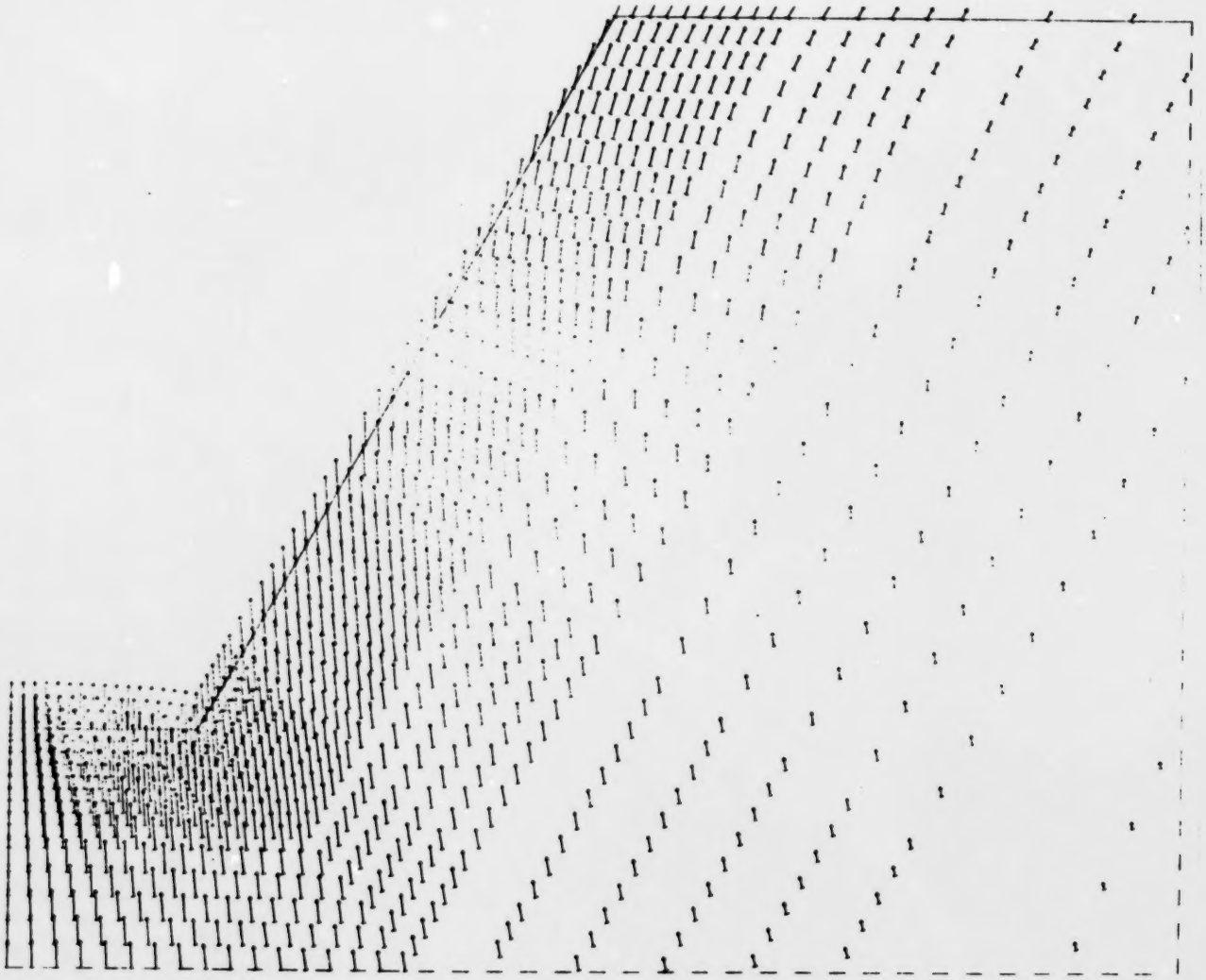


Figure 12. Excavation Displacement Vector Plot in Critical Region.
(Model Scale : 1 cm = 27.0 m)

STRESS OUTPUT FOR 2-D PLANE OR AXISYMMETRIC ELEMENTS
CENTRE STRESSES IN Y-Z COORDS, BOUNDARY STRESSES NORMAL AND PARALLEL TO THE SIDES

EL.NO.	LOAD	FACE	SIG-YY SIG-RR	SIG-ZZ	SIG-XX SIG-TT	SIG-YZ SIG-RZ	S-MAX	S-MIN	ANGL
1	1	CEN	5.138672E+01	-6.284375E+01	5.138672E+01	-2.837891E+01	5.804853E+01	-6.950557E+01	-13.211
2	1	CEN	-4.761367E+02	-2.272344E+02	-4.761367E+02	-4.192188E+01	-2.203633E+02	-4.830078E+02	-80.692
3	1	CEN	-9.530664E+02	-4.308125E+02	-9.530664E+02	-5.448047E+01	-4.251897E+02	-9.586892E+02	-84.108
4	1	CEN	-1.382934E+03	-6.715469E+02	-1.382934E+03	-6.864844E+01	-6.649829E+02	-1.389498E+03	-84.538
5	1	CEN	-1.764750E+03	-9.494063E+02	-1.764750E+03	-8.324609E+01	-9.409937E+02	-1.773163E+03	-84.230
6	1	CEN	-2.099188E+03	-12.61766E+03	-2.099188E+03	-9.734375E+01	-12.50599E+03	-2.110354E+03	-83.456
7	1	CEN	-2.388574E+03	-1.603641E+03	-2.388574E+03	-1.100625E+02	-1.588500E+03	-2.403715E+03	-82.168
8	1	CEN	-2.636563E+03	-1.968875E+03	-2.636563E+03	-12.08633E+02	-1.947670E+03	-2.657768E+03	-80.049
9	1	CEN	-2.847621E+03	-2.350813E+03	-2.847621E+03	-12.95273E+02	-2.319070E+03	-2.879363E+03	-76.231
10	1	CEN	-3.026703E+03	-2.743500E+03	-3.026703E+03	-1.359082E+02	-2.688831E+03	-3.081372E+03	-68.088
11	1	CEN	-3.239268E+03	-3.354469E+03	-3.239268E+03	-1.416113E+02	-3.143990E+03	-3.449746E+03	-33.933
12	1	CEN	-3.457111E+03	-4.142992E+03	-3.457111E+03	-1.427930E+02	-3.428571E+03	-4.171533E+03	-11.303
13	1	CEN	-3.619188E+03	-4.901086E+03	-3.619188E+03	-1.389922E+02	-3.604290E+03	-4.915983E+03	-6.118
14	1	CEN	-3.745654E+03	-5.617195E+03	-3.745654E+03	-1.322168E+02	-3.736360E+03	-5.626490E+03	-4.021
15	1	CEN	-3.850229E+03	-6.287645E+03	-3.850229E+03	-12.39609E+02	-3.843940E+03	-6.293933E+03	-2.904
16	1	CEN	-4.011859E+03	-7.301637E+03	-4.011859E+03	-1.087451E+02	-4.008354E+03	-7.305143E+03	-1.846
17	1	CEN	-4.210573E+03	-8.709961E+03	-4.210573E+03	-8.957813E+01	-4.208791E+03	-8.711744E+03	-1.140
18	1	CEN	-4.403521E+03	-9.871729E+03	-4.403521E+03	-7.442773E+01	-4.402509E+03	-9.872741E+03	-0.780
19	1	CEN	-4.598967E+03	-1.091714E+04	-4.598967E+03	-6.278125E+01	-4.598344E+03	-1.091776E+04	-0.569
20	1	CEN	-4.798463E+03	-1.188075E+04	-4.798463E+03	-5.382471E+01	-4.798054E+03	-1.188116E+04	-0.435
21	1	CEN	-5.120860E+03	-1.326895E+04	-5.120860E+03	-4.371680E+01	-5.120625E+03	-1.326919E+04	-0.307
22	1	CEN	-5.557904E+03	-1.498470E+04	-5.557904E+03	-3.435645E+01	-5.557779E+03	-1.498483E+04	-0.209
23	1	CEN	-6.003829E+03	-1.660556E+04	-6.003829E+03	-2.781519E+01	-6.003755E+03	-1.660563E+04	-0.150
24	1	CEN	-6.455529E+03	-1.816649E+04	-6.455529E+03	-2.295972E+01	-6.455484E+03	-1.816654E+04	-0.112
25	1	CEN	-6.911126E+03	-1.968719E+04	-6.911126E+03	-1.917505E+01	-6.911098E+03	-1.968722E+04	-0.086
26	1	CEN	-7.369464E+03	-2.117951E+04	-7.369464E+03	-1.609521E+01	-7.369446E+03	-2.117953E+04	-0.067
27	1	CEN	-7.829819E+03	-2.265093E+04	-7.829819E+03	-1.348560E+01	-7.829808E+03	-2.265094E+04	-0.052
28	1	CEN	-8.291725E+03	-2.410637E+04	-8.291725E+03	-1.119019E+01	-8.291717E+03	-2.410638E+04	-0.041
29	1	CEN	-8.754866E+03	-2.554917E+04	-8.754866E+03	-9.096985E+01	-8.754861E+03	-2.554917E+04	-0.031
30	1	CEN	-9.219034E+03	-2.698160E+04	-9.219034E+03	-7.114532E+01	-9.219031E+03	-2.698160E+04	-0.023
31	1	CEN	-9.684086E+03	-2.840522E+04	-9.684086E+03	-5.167603E+01	-9.684086E+03	-2.840522E+04	-0.016
32	1	CEN	-1.014991E+04	-2.982106E+04	-1.014991E+04	-3.183334E+01	-1.014991E+04	-2.982106E+04	-0.009
33	1	CEN	-1.061642E+04	-3.122982E+04	-1.061642E+04	-1.998862E+00	-1.061642E+04	-3.122982E+04	-0.003
34	1	CEN	2.092969E+01	-6.365625E+01	3.485938E+01	-1.636719E+01	2.398625E+01	-6.671281E+01	-10.578
35	1	CEN	-4.997930E+02	-2.280156E+02	-4.889922E+02	-4.928906E+01	-2.193528E+02	-5.084558E+02	-80.032
36	1	CEN	-9.801836E+02	-4.314219E+02	-9.677578E+02	-8.455078E+01	-4.186900E+02	-9.929155E+02	-81.437
37	1	CEN	-1.412242E+03	-6.747188E+02	-1.399199E+03	-1.235977E+02	-6.545569E+02	-1.432404E+03	-80.736
38	1	CEN	-1.794359E+03	-9.570000E+02	-1.781859E+03	-1.640625E+02	-9.260029E+02	-1.825356E+03	-79.301
39	1	CEN	-2.127215E+03	-1.275203E+03	-2.116352E+03	-2.032891E+02	-1.229184E+03	-2.173234E+03	-77.245
40	1	CEN	-2.413344E+03	-1.623828E+03	-2.405012E+03	-2.391133E+02	-1.557057E+03	-2.480115E+03	-74.398
41	1	CEN	-2.656813E+03	-1.996016E+03	-2.651645E+03	-2.699883E+02	-1.899733E+03	-2.753095E+03	-70.373
42	1	CEN	-2.862598E+03	-2.384781E+03	-2.860910E+03	-2.949414E+02	-2.244127E+03	-3.003252E+03	-64.504
43	1	CEN	-3.036148E+03	-2.783734E+03	-3.037977E+03	-3.137090E+02	-2.571797E+03	-3.248086E+03	-55.958
44	1	CEN	-3.241090E+03	-3.402453E+03	-3.247633E+03	-3.319727E+02	-2.980135E+03	-3.663408E+03	-38.170
45	1	CEN	-3.450410E+03	-4.197258E+03	-3.461852E+03	-3.386465E+02	-3.319724E+03	-4.327944E+03	-21.102
46	1	CEN	-3.606449E+03	-4.957805E+03	-3.621055E+03	-3.318984E+02	-3.529334E+03	-5.034920E+03	-13.080
47	1	CEN	-3.729174E+03	-5.673609E+03	-3.745459E+03	-3.171387E+02	-3.678756E+03	-5.724028E+03	-9.033
48	1	CEN	-3.831637E+03	-6.341984E+03	-3.848580E+03	-2.982324E+02	-3.796693E+03	-6.376928E+03	-6.683
49	1	CEN	-3.993102E+03	-7.429947E+03	-4.009191E+03	-2.624727E+02	-3.973172E+03	-7.449877E+03	-4.342
50	1	CEN	-4.193507E+03	-8.749484E+03	-4.207464E+03	-2.169053E+02	-4.183204E+03	-8.759788E+03	-2.720

Figure 13. Sample of SAP2D Printer Stress (kPa) Output.

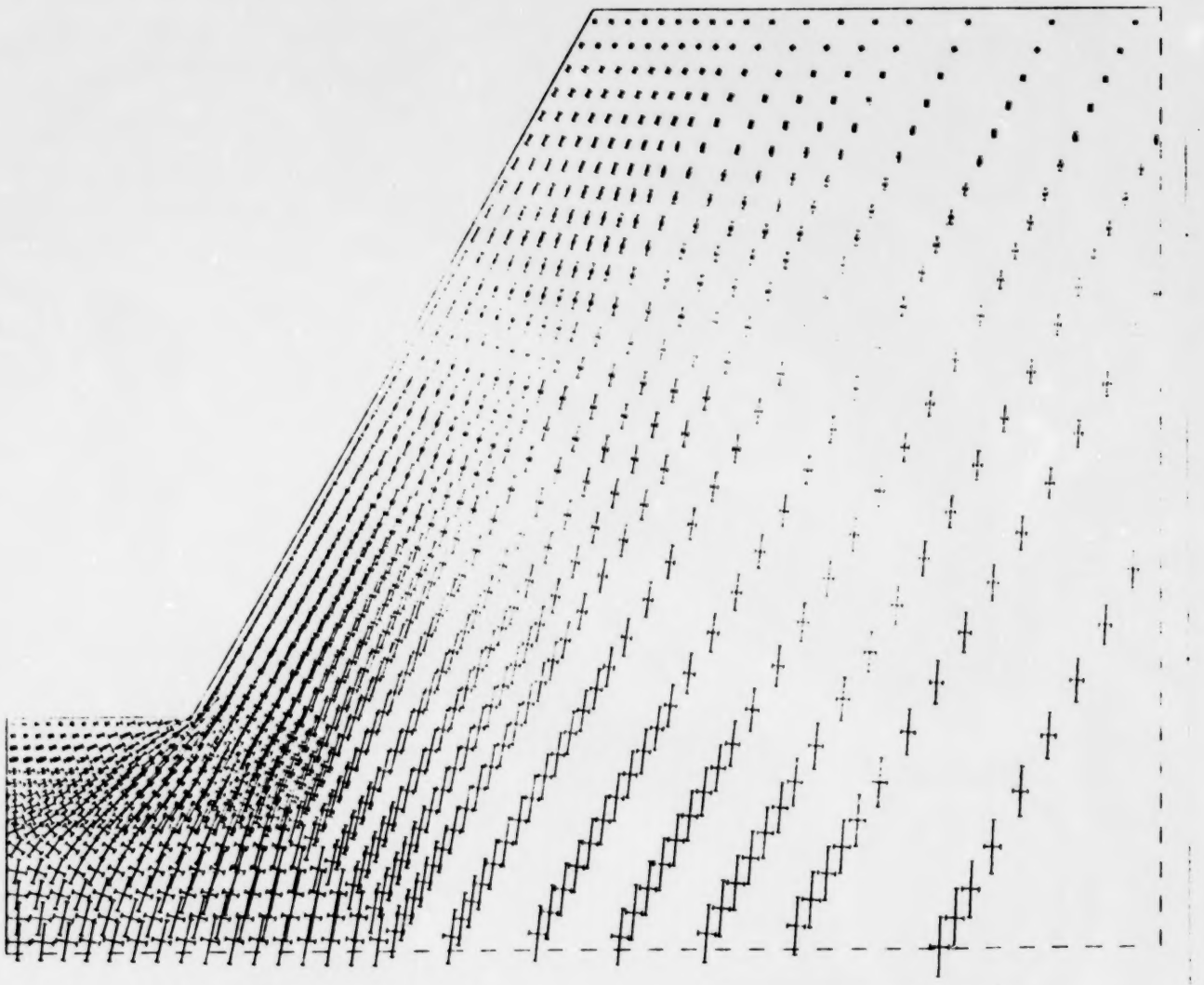


Figure 14. Principal Stresses Plotted as Vectors in Critical Region.
(Model Scale : 1 cm = 27.0 m)

To avoid this problems point contour plots of the major and minor principal stresses can be made. Figure 15 and 16 give examples of these for the slope problem.

More frequently we are interested in some scalar function of the stress that is related to local stability, *e.g.*, local safety factors or some yield function. Figure 17 is an example of such a plot over the critical region of the slope. Here the Drucker – Prager yield function has been plotted with assumed values of cohesion and internal friction. Although the numbers produced may not be too meaningful we still get a convenient approximate ordering of our structure into stability zones.

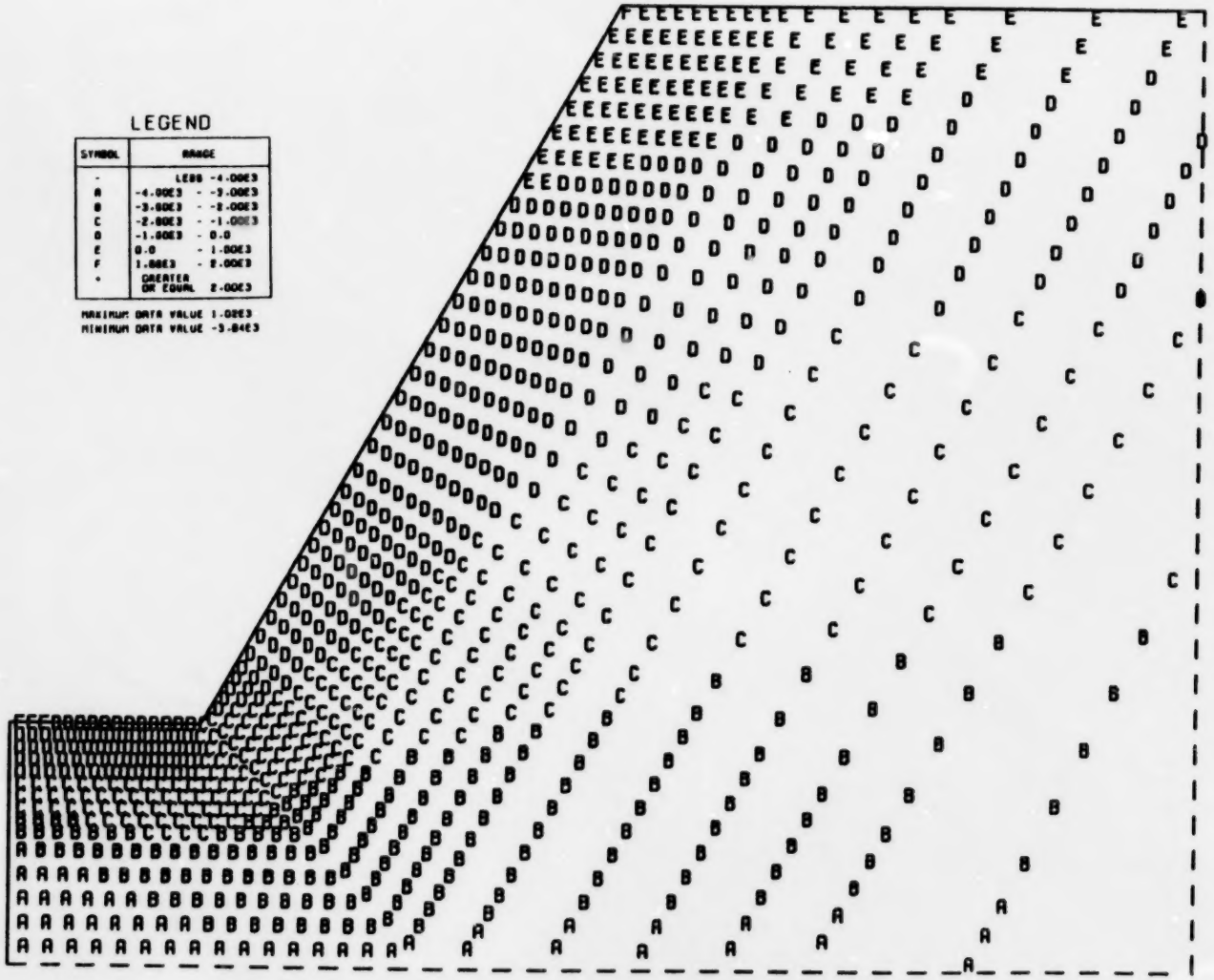


Figure 15. Major Principal Stress Point Contour Plot in Critical Region. (Model Scale : 1 cm = 27.0 m)

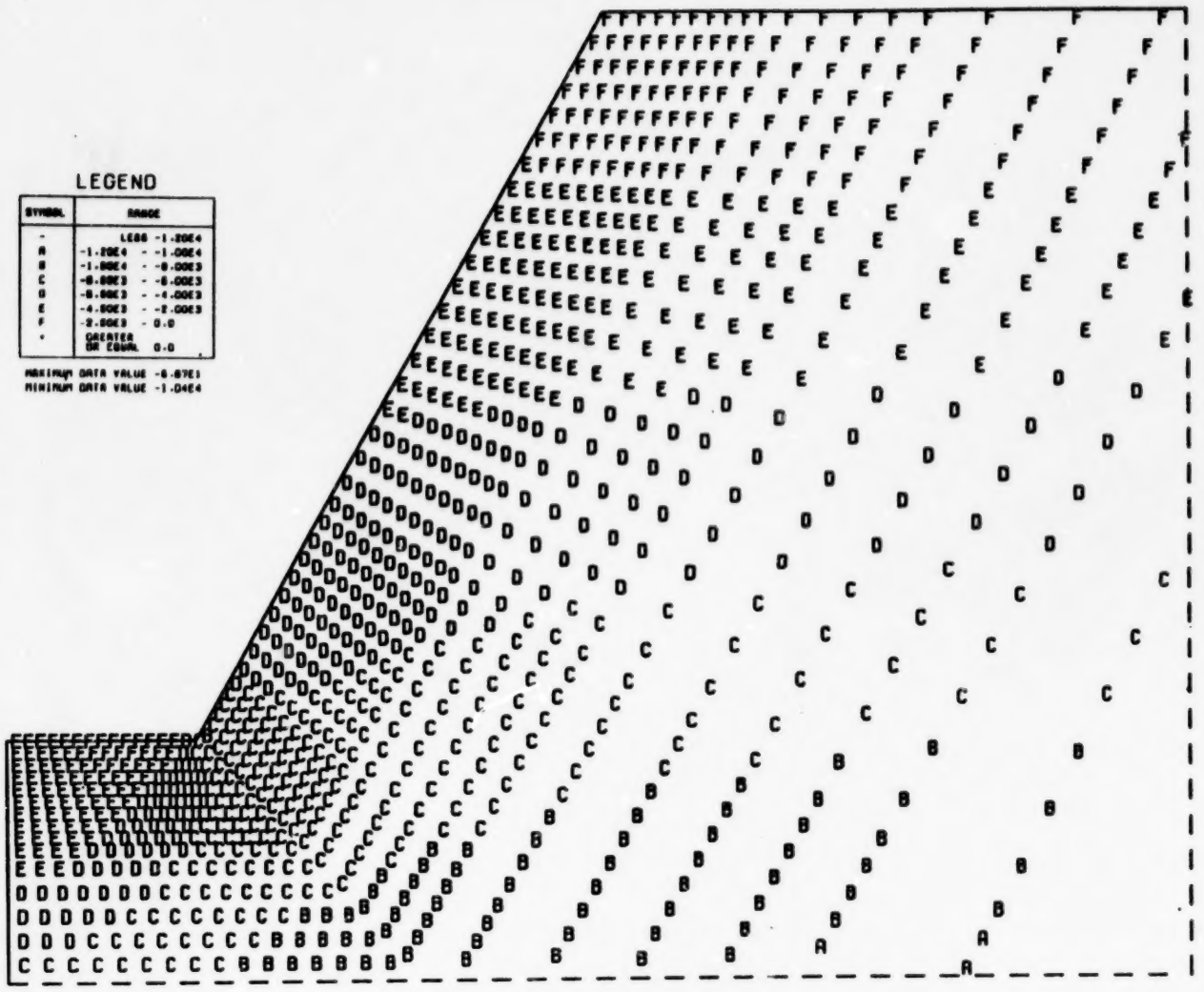


Figure 16. Minor Principal Stress Point Contour Plot in Critical Region. (Model Scale : 1 cm = 27.0 m)

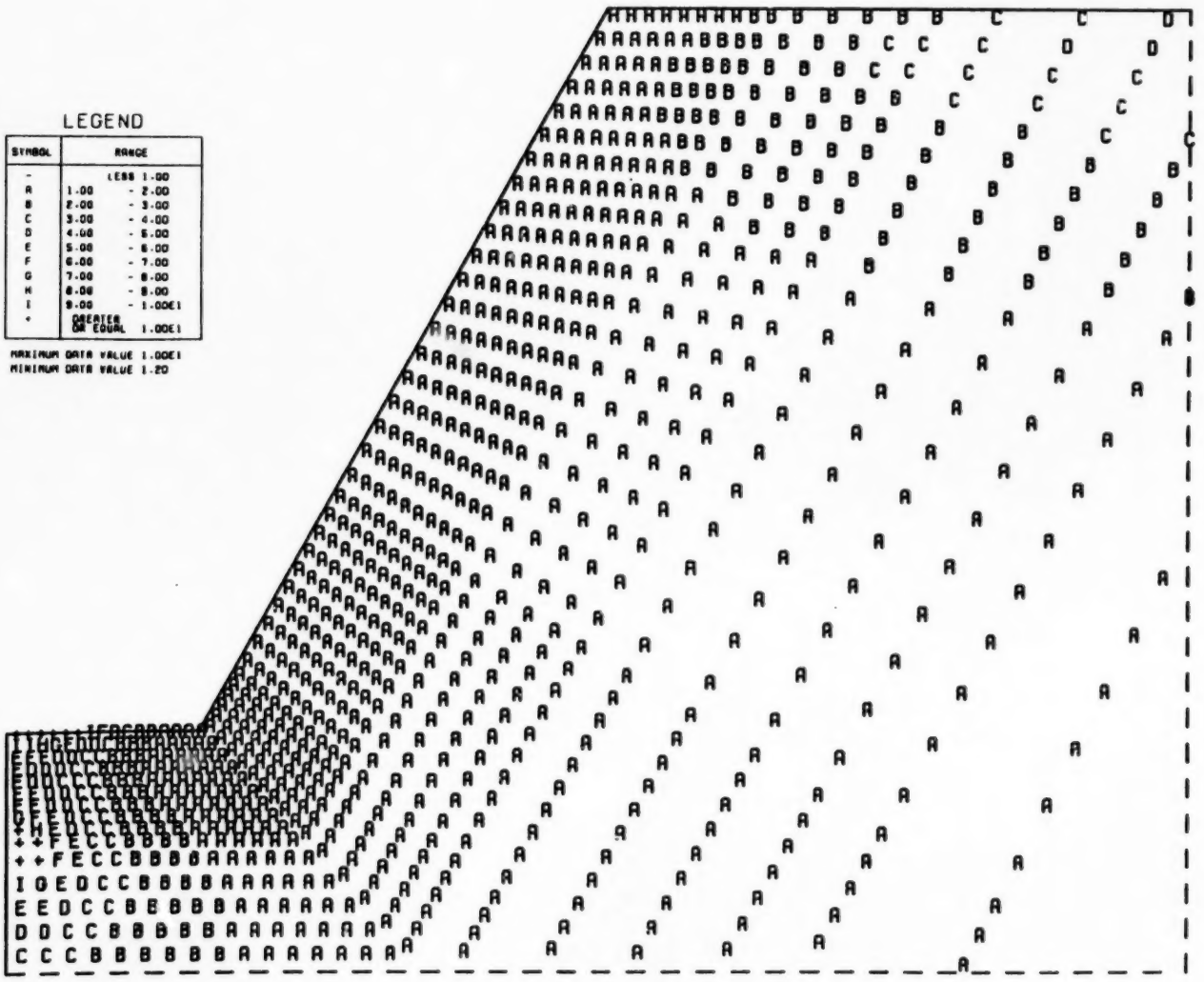


Figure 17. Drucker/Prager Stress Function Point Contour Plot in Critical Region. (Cohesion = 1000 kPa, Internal Friction Angle 35 Degrees and Model Scale : 1 cm = 27.0 m)

14.0 REFERENCES

1. Wilson, E.L.; SOLID SAP – A static analysis program for three dimensional solid structures; UC SESM 71–79 University of California, Berkeley, U.S.A.; (1972).

2. Toews, N.A. and Yu, Y.S.; SAP2D documentation - 2D Linear elastic finite element computer System; Report 74/138; Mining Research Laboratories, CANMET, Energy Mines and Resources Canada; Ottawa; December 1974.

3. Toews, N.A., Yu, Y.S. and Wong, A.S.; MSHGEN documentation (VAX-11/750 version) – A mesh generating program for finite element analysis; Division Report MRP/MRL 85-32 (TR); Mining Research Laboratories, CANMET, Energy Mines and Resources Canada; Ottawa; February 1985.

4. Yu, Y.S., Toews, N.A. and Wong, A.S.; MSHPLT documentation (VAX-11/750 version) – A mesh plotting program for finite element analysis; Division Report MRP/MRL 85-33 (TR); Mining Research Laboratories, CANMET, Energy Mines and Resources Canada; Ottawa; February 1985.

5. Yu, Y.S., Toews, N.A. and Wong, A.S.; SAP2DC documentation (VAX-11/750 version) – An interface program for the finite element program SAP2D; Division Report MRP/MRL 85-34 (TR); Mining Research Laboratories, CANMET, Energy Mines and Resources Canada; Ottawa; February 1985.

Appendix A. – Nodal Point Generation

If the nodal structure is sufficiently regular it may be possible to generate certain nodal points using the simple SAP2D nodal point generator. This generator works as follows. Suppose that two consecutive cards are read the first with nodal number NOLD and with co-ordinates Y(NOLD) and Z(NOLD) and the second card with nodal number N, co-ordinates Y(N) and Z(N) and generation parameter KN. If KN is less than or equal to zero no generation occurs. If KN is positive then NUM nodes will be generated (if NUM is positive) where NUM is the largest integer such that NOLD + NUM × KN is less than N. The generated points lie equally spaced on the line joining [X(NOLD), Y(NOLD)] and [X(N), Y(N)] and are given nodal numbers NOLD + KN, NOLD + 2 × KN, ... , NOLD + NUM × KN respectively. The generated nodal points have unconstrained displacements unless one or both of the constraint indicators at NOLD is negative, in which case the corresponding generated displacement constraint will also be negative.

Appendix B. – Angle of Anisotropy

In isotropic case :

EN = ET = E;
RNUNS = RNUNT = RNUST = RNU;
GNS = E / 2 (1 + RNU); and
HAN = HAS = HAT = HA.

In the general case orthotropic temperature dependent material properties are allowed. The cards read in above define the properties versus temperature table. Assuming that nodal point temperatures have been inputed the element temperature is found by averaging the temperature over the element associated nodal points. The appropriate mechanical properties of this element are then calculated by doing a linear interpolation over the defined mechanical properties-temperature tables. The graph of mechanical properties versus temperature is assumed to be constant at temperatures lower than the lowest tabulated temperatures and temperatures higher than the highest tabulated temperatures respectively. Figure 18 defines the principal material axis N and S of the anisotropic material. The direction T is out of the paper.

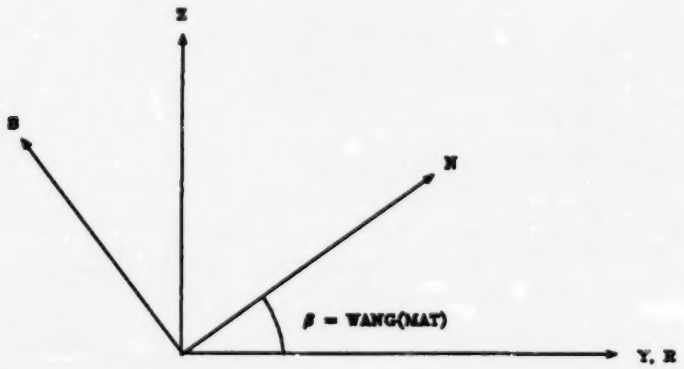


Figure 18. Principal Material Axis N–S.

Appendix C. – Quadrilateral Element Generation

Suppose that in the previous card read contained NOLD, IO, JO, KO, LO, MATO, NUMPCO, NSO, KGO, TO, THICKO and that current card contains NNEW, IN, JN, KN, LN, MATN, NUMPCN, NSN, KGN, TN, THICKN then if NNEW is greater than NOLD + 1 then all information for element NOLD + 1, ..., NNEW – 1 is generated as follows. Consider element NOLD + M = N then the card generated for this is N, I, J, K, L, MATO, NCUTO, NUMPCO, NSO, KG, THICKO where

$$
\begin{aligned}
I &= IOLD + M \times KGN \text{ (Note : KGN not KGO is used for generation)} \\
J &= JOLD + M \times KGN \\
K &= KOLD + M \times KGN \\
L &= LOLD + M \times KGN
\end{aligned}
$$

KG is set to zero to denote a generated element. Distributed loads (if any) are assumed identical to that of previous card.

Appendix D. – Boundary Element Generation

(a) In many problems a given displacement constraint direction remains fixed for a number of elements. To avoid having to introduce (possibly artificial) direction nodes for all these elements the following generation feature has been built in. If the direction node is zero then the direction defined by the last previous card with a non-zero entry is used.

(b) Omitted elements are also generated. In this case the location nodes of generated elements are found by incrementing the location node of the previous card by increasing multiples of the element generation parameter of the current card. Element direction, cut number, specified displacement and axial stiffness are all as on the previous card.

APPENDIX E -- SAP2D COMMAND PROCEDURE

```
$ ! SAP2D COMMAND PROCEDURE
$ !
$ WRITE SYS$OUTPUT "YOU ARE RUNNING THE SAP2D COMMAND PROCEDURE"
$ !
$AGAIN1:
$ WRITE SYS$OUTPUT " "
$ WRITE SYS$OUTPUT "INPUT MSHGEN INPUT FILE"
$ INQUIRE MSHIN "DEFAULT - MSHGEN.DAT"
$ IF MSHIN.EQS."" THEN MSHIN := "MSHGEN.DAT"
$ MSHIN = F$SEARCH(MSHIN,1)
$ IF MSHIN.NES."" THEN GOTO AGAIN2
$ WRITE SYS$OUTPUT "SPECIFIED FILE DOES NOT EXIST"
$ GOTO AGAIN1
$ !
$AGAIN2:
$ WRITE SYS$OUTPUT " "
$ WRITE SYS$OUTPUT "INPUT SAP2DC INPUT FILE"
$ INQUIRE SAP2DCIN "DEFAULT - SAP2DC.DAT"
$ IF SAP2DCIN.EQS."" THEN SAP2DCIN := "SAP2DC.DAT"
$ SAP2DCIN = F$SEARCH(SAP2DCIN,1)
$ IF SAP2DCIN.NES."" THEN GOTO AGAIN3
$ WRITE SYS$OUTPUT "SPECIFIED FILE DOES NOT EXIST"
$ GOTO AGAIN2
$ !
$AGAIN3:
$ WRITE SYS$OUTPUT " "
$ WRITE SYS$OUTPUT "INPUT SAP2D OUTPUT FILE"
$ INQUIRE SAP2DOUT "DEFAULT - SAP2DOUT.DAT"
$ IF SAP2DOUT.EQS."" THEN SAP2DOUT := "SAP2DOUT.DAT"
$ SAP2DOUT = F$PARSE(SAP2DOUT,,,,"SYNTAX_ONLY")
$ IF SAP2DOUT.NES."" THEN GOTO AGAIN4
$ WRITE SYS$OUTPUT "BAD FILE NAME"
$ GOTO AGAIN3
$ !
$AGAIN4:
$ !
$! LOGNAM = F$EXTRACT(0,F$LOCATE(";",SAP2DOUT)-3,SAP2DOUT)
$ LOGNAM := "SAP2DLOG."
$ !
$ SUBMIT/PARAMETERS=("''MSHIN'","''SAP2DCIN'","''SAP2DOUT'")-
 /LOG_FILE='LOGNAM'LOG    DISK$USER:[FE.COMMAND]SAP2.COM
```

APPENDIX F -- SAP2D SUB-COMMAND PROCEDURE

```
$ ! COMMAND PROCEDURE SAP2.COM
$ !
$ !
$ ON ERROR THEN GOTO FINISH
$ ON CONTROL_Y THEN GOTO FINISH
$ !
$ ASSIGN 'P1' FOR005
$ ASSIGN GENOUTP.DUM GENOUT
$ ASSIGN NL: FOR006
$ ! RUN MSHGEN
$ RUN DISK$USER:[FE.EXECUTE]MSHGEN
$ !
$ ASSIGN 'P2' FOR005
$ ASSIGN SAPINP.DUM SAPIN
$ ASSIGN SYS$OUTPUT FOR006
$ ! RUN SAP2DC
$ RUN DISK$USER:[FE.EXECUTE]SAP2DC
$ ASSIGN SAPINP.DUM FOR005
$ ASSIGN 'P3' SAPOUT
$ DEVICE=F$PARSE("''P3'",,,"DEVICE")
$ DEV_CLASS=F$GETDVI("''DEVICE'","DEVCLASS")
$ IF DEV_CLASS .NE. 2 THEN GOTO NEXT
$ LABEL=F$PARSE("''P3'",,,"NAME")
$ MOUNT 'DEVICE' 'LABEL'
$NEXT:
$ RUN DISK$USER:[FE.EXECUTE]SAP2D
$ !
$FINISH:
$ SET NOON
$ DELETE GENOUTP.DUM;*
$ DELETE SAPINP.DUM;*
$ IF DEV_CLASS .EQ. 2 THEN DISMOUNT 'DEVICE'
$ SET ON
```